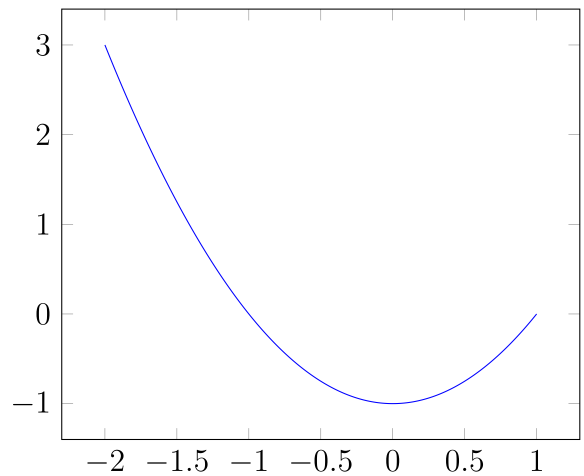
Math 10A
Worksheet, Discussion #4; Thursday, 6/21/2018
Instructor name: Roy Zhao

1 Transforming Functions

1.1 Concepts

1. Vertical stretching and shifting is what is done to $f(x)$. Multiplying by a constant greater than 1 stretches the graph and adding a positive number shifts the graph up. Horizontal stretching and shifting is what is done to the x inside $f(x)$. Multiplying by a constant greater than 1 compresses the graph and adding a positive number shifts the graph to the left. We treat the order of shifting and stretching opposite from the vertical case.

1.2 Example

2. Let $f(x)$ be the function shown in the graph 

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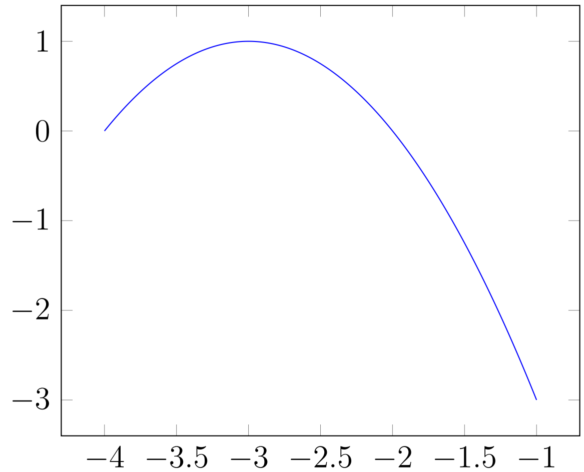
Draw and find the domain and range of $-f(-x-3)$.

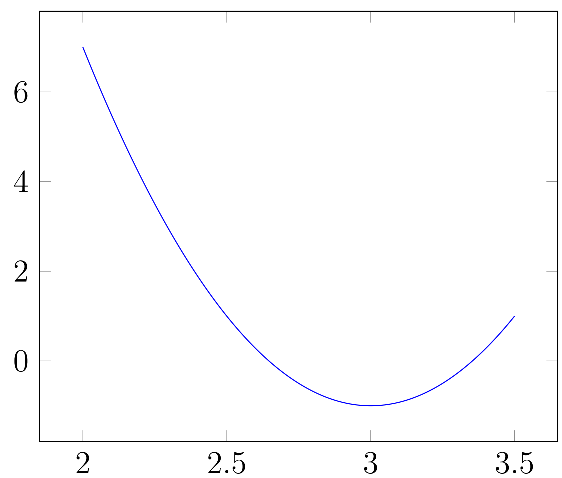
Solution: For the domain, the original domain was $[-2, 1]$, then looking at the -3 we shift it right by 3 then the $-x$ tells us to reflect it so we get $[1, 4]$, then $[-4, -1]$ for the final domain. For the range, we simply reflect to go from $[-1, 3]$ to $[-3, 1]$.



1.3 Problems

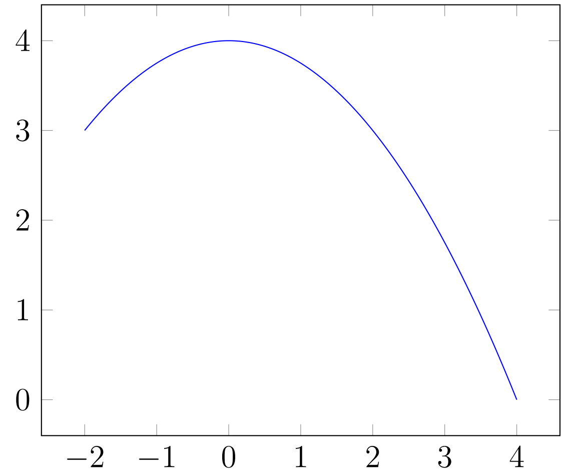
3. Using the same function from before, draw and find the domain and range of $2f(2x - 6) + 1$.

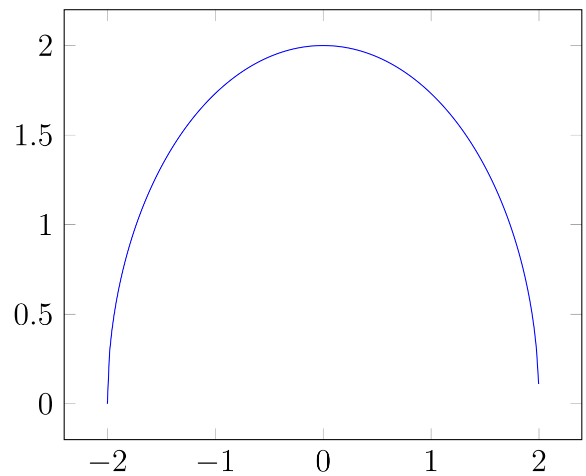
Solution: For the domain, we add 6 for the -6, then we divide by 2 for the $2x$ to get $[-2, 1] \to [4, 7] \to [2, 3.5]$. For the range, we first multiply by 2 then add 1 to get $[-1, 3] \to [-2, 6] \to [-1, 7]$.



4. Using the same function from before, draw and find the domain and range of $-f(-x/2) + 3$.

Solution: For the domain, we multiply by -2 for the $-x/2$ to get $[-2, 1] \to [-2, 4]$. For the range, we first multiply by -1 then add 3 to get $[-1, 3] \to [-3, 1] \to [0, 4]$.



5. Let $g(x)$ be the function shown in the graph 

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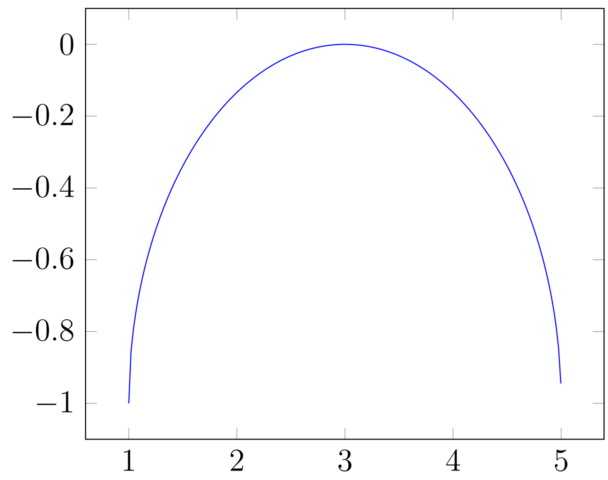
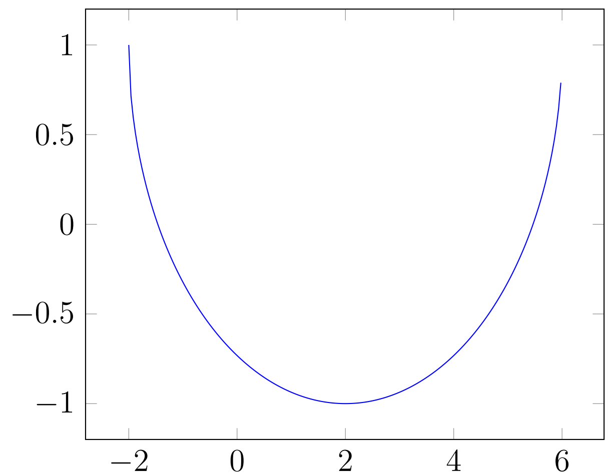
Draw and find the domain and range of $g(-x + 3)/2 - 1$.

Solution: For the domain, we subtract 3 for the $+3$, then we multiply by -1 for the $-x$ to get $[-2, 2] \to [-5, -1] \to [1, 5]$. For the range, we first divide by 2 then subtract 1 to get $[0, 2] \to [0, 1] \to [-1, 0]$.



6. Using the same function from before, draw and find the domain and range of $-g(1 - x/2) + 1$.

Solution: For the domain, we subtract 1 for the +1, then we multiply by −2 for the $-x/2$ to get $[-2, 2] \to [-3, 1] \to [-2, 6]$. For the range, we first multiply by −1 then add 1 to get $[0, 2] \to [-2, 0] \to [-1, 1]$.



7. Write the function that is $\sqrt{x}$ shifted to the left by 3 then horizontally stretched by 5. Then compressed vertically by a factor of 4 and shifted down by 1.

Solution: The horizontal transformation tells us that x is first divided by 5 then 3 is added. The vertical transformation tells us that the function is divided by 4 then 1 is subtracted. So the function is $\sqrt{x/5 + 3}/4 - 1$.

8. Write the function that is $1/x$ shifted to the right by 2 then horizontally compressed by 3 and reflected. Then stretched vertically by a factor of 2 and shifted down by 4.

Solution: The horizontal transformation tells us that x is first multiplied by -3 then 2 is subtracted. The vertical transformation tells us that the function is multiplied by 2 then 4 is subtracted. So the function is $\dfrac{2}{-3x-2} - 4$.